A Simple Climate Model

How can you predict what the average temperature of a planet should be?

Read This!

You will need to calculate some things. Get your calculators out! You will need to know the formula for the area of a circle. You will need to know the formula for the surface area of a sphere. You will also be required to find the fourth-root $\left(\sqrt[4]{x}\right)$ of a number with your calculator.

Required Skills

1. Using your calculator, show that you can calculate the following :

5.67 × 10⁻⁸ J/(m²K⁴) × **4** × **π** = ***7.13 × 10⁻⁷*** J/(m²K⁴)

$\sqrt[4]{1280}$ = ***5.981***

Make sure that you can solve for fourth-roots before moving on. Show your teacher.

Read This!

You will also need to be able to convert between temperature units. To convert from Kelvin (K) to degrees Celsius (°C), you subtract 273.15. To convert from degrees Celsius (°C) to Kelvin (K), you add 273.15.

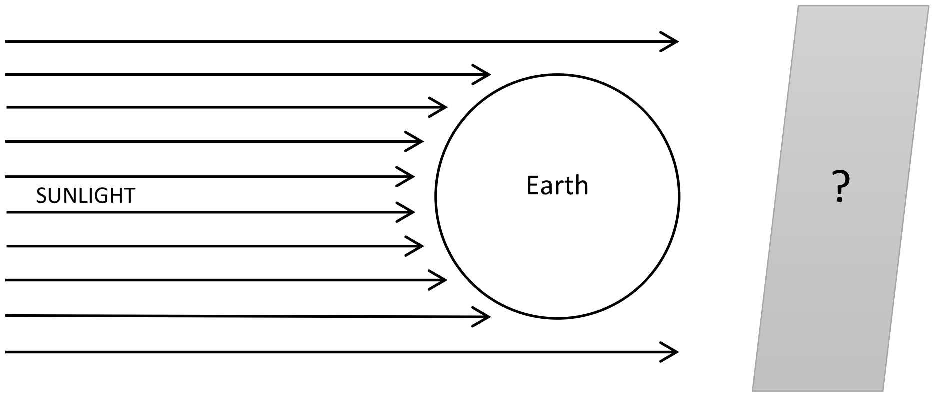
2. In Kelvin, what is the current air temperature of the room you are in? Convert your room's temperature from °C to K.

   If the current temperature is 20°C, that is ***293.15 K***.

3. The sun's surface temperature is 5.777×10^3 K. What temperature is that in degrees Celsius?

   5.504×10^3 °C

Model 1 – Earth casts a shadow



4. What is the shape of the shadow that the Earth casts?

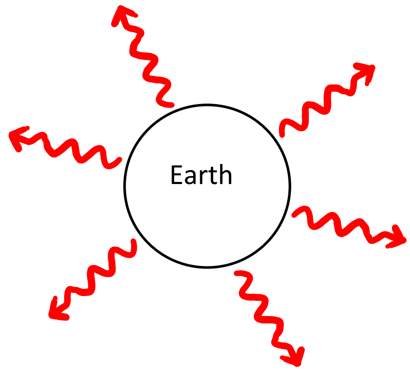
   (a) an oval   ***(b) a circle***   (c) a square

5. The radius of the Earth is **6.37×10^6 m**. How big of a shadow does the Earth cast? What is the area of its shadow? Show your calculation and give your answer in the units: m^2.

   area of a circle = πR^2

   = $(3.14)(6.37 \times 10^6 \text{ m})^2$
   = ***1.27×10^{14} m^2***

Model 2 – The Earth loses heat to outer space in all directions



Read This!

The Earth radiates its heat into space over its whole surface. The side facing the sun and the side not facing the sun both radiate infrared energy into space. This happens because the surface temperature of the Earth is higher than the temperature of outer space. Even the coldest places on Earth are warmer than outer space.

6. The Earth is a large sphere that loses heat energy over its surface.
   What is the formula to calculate the surface area of a sphere?

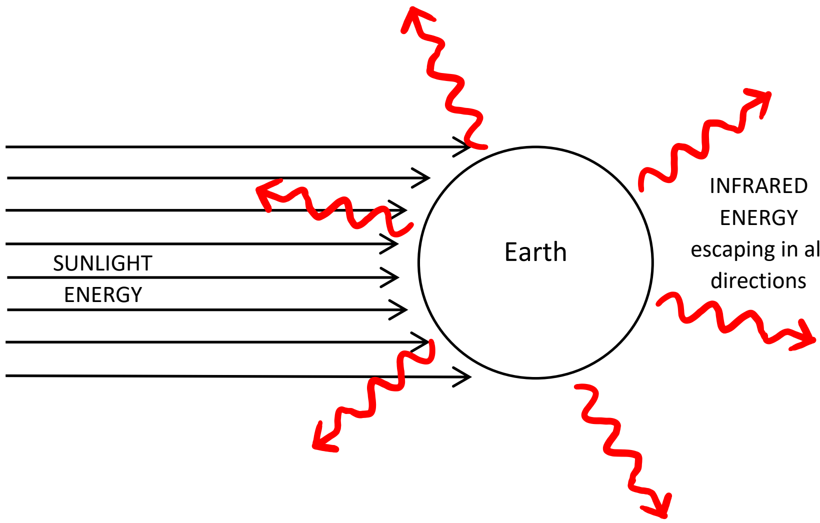
   surface area of a sphere = $\mathbf{4\pi r^2}$

Make sure that your teacher confirms that you have the correct formula to calculate the surface area of a sphere.

7. Calculate the Earth's surface area using the formula from question 6. Show your calculation and give your answer in the units: m^2.

   $$\mathbf{4\pi r^2} = (\mathbf{4})(\mathbf{3.14})(\mathbf{6.37} \times \mathbf{10^6})^2 = \mathbf{5.10 \times 10^{14}\ m^2}$$

Model 3 –
The Earth blocks sunlight like a circle but loses heat like a sphere



Read This!

Model 3 represents an "Energy Balance Climate Model". Things (like planets) orbiting a star are said to be in *thermal equilibrium*. That means that amount of sunlight energy striking the planet must equal the amount of infrared energy that the planet loses to outer space.

$$Sunlight\ Energy_{\ in} = Infrared\ Energy_{\ out}$$

8. From the list below, circle all of the things that you think would affect the amount of sunlight energy that a planet absorbs.

   (a) the energy output of the star

   (b) the distance the planet is from the star

   (c) the radius of the planet

   (d) the albedo of the planet

9. Do you think that the radius of a planet would affect the amount of infrared energy that it loses to outer space? Explain why.

   Yes. The greater the radius of a planet, the faster it should cool because there is a larger surface area for heat exchange to happen between the earth and outer space.

10. A hotter planet should lose heat to outer space ***faster*** than a colder planet would.

11. Do you think that a planet's average surface temperature would affect the amount of infrared energy that it loses to outer space?

    YES NO (circle one)

12. Explain the reasons why you circled yes, or no, in question #11.

    A hot cup of tea loses heat faster than warm cup of tea.
    For the same reason, a greater amount of heat (infrared) leaves a planet with a higher temperature.

How much sunlight energy reaches the Earth?

At the Earth's distance from the sun, the energy from the sun that is streaming through the solar system in all directions is: **1350 Joules of energy, per m^2, per second.**

13. Would the Earth's albedo affect the amount of sunlight energy that the Earth absorbs?

    Yes.

14. If the Earth's albedo went UP, would the amount of sunlight energy that the Earth absorbed go up, or would the amount of sunlight energy absorbed go down?

    The earth would absorb LESS sunlight energy.

15. Would the radius of the Earth affect the amount of sunlight energy that the Earth absorbed? Why, or why not?

    Yes, Earth's radius would affect the amount of sunlight that Earth absorbed. The larger the radius of the earth, the more sunlight it would block (and absorb).

Albedo (α) – has a number!

Albedo (how much sunlight is reflected) is expressed on a scale of zero to one.
If the Earth **reflected all** the sunlight that hit it, the Earth would have an albedo of "1".
However, if the Earth **absorbed all** of the sunlight that hit it, it would have an albedo of "0".

The Earth's clouds and ice prevent approximately 1/3 of the sun's energy from reaching and heating the surface. For the Earth, **α = 0.3**.

To find the proportion of the sun's energy that is absorbed, you calculate **1 – α**.
In Earth's case, this means that approximately 70% of the available sunlight energy is absorbed somewhere around the planet.

16. Using the radius of the Earth (given in question #5) and the following equation, calculate the amount of sunlight absorbed by the Earth every second.

$$sunlight\ energy_{in} = (1-\alpha) \times circular\ area\ of\ Earth\ \times 1350\ J/m^2$$

The Earth absorbs = (1 – 0.3) * π * $(6.37 \times 10^6\ m)^2 \times 1350\ J/m^2$ = ***1.20×10^{17} J*** every second.

STOP Check with your teacher to ensure that you have calculated the answer correctly.

How much infrared energy radiates back out into space?

Our energy balance model, Model 3, assumes that a planet reaches *thermal equilibrium*, after a period of time:

$$Sunlight\ Energy_{\ in} = Infrared\ Energy_{\ out}$$

You have already calculated what the sunlight energy "in" is. That value is based on the Sun's energy output, the Earth's albedo, and the Earth's circular area.

17. Based on the equation above, what must the amount of Earth's outgoing infrared energy be every second?
(Your answer should be in Joules, and it should be the same as your answer from question #16.)

$Infrared\ Energy_{out} =$ ***1.20×10^{17} J***

What should the average temperature of the Earth be?

The infrared energy that radiates from objects can be calculated using the Stefan-Boltzmann Law.

The energy that leaves a planet as it cools, depends on these three things:
① - the surface area of the planet
② - the Stefan-Boltzmann constant
③ - the average (or 'skin') temperature of the planet [raised to the power of 4]

When applied to our planet (the Earth), the word equation for the Stefan-Boltzmann Law looks like this:

$$Energy_{out} = (surface\ area\ of\ Earth) \times (Stefan\ Boltzmann\ constant) \times (average\ temperature)^4$$

As an equation, it looks like this:

$$Infrared\ Energy_{out} = 4\pi r^2 \times \sigma \times T^4$$

18. In the space below, please rearrange the above equation so that average temperature (*T*) is alone on the left hand side of the equation.

$$\boldsymbol{T = \sqrt[4]{\frac{Infrared\ Energy_{out}}{4\pi r^2 \times \sigma}}}$$

19. Calculate what the temperature of Earth should be according to Model 3, using the equation from question #18.
    - the Stefan-Boltzmann constant (σ) equals **5.67 x 10^{-8} J/(m^{2}K^4)**
    - the radius of the Earth was given in question #5.

    Your answer should have the temperature unit **Kelvin** or **K**. Show your work.

    254 K

20. Convert your answer from question #19 from Kelvin (K) to degrees Celsius (°C).

    -19 °C

21. When measured, the actual average temperature of the Earth turns out to be about **15 °C**. How many degrees hotter is this than the temperature calculated in question #20?

    34 °C

Read This!

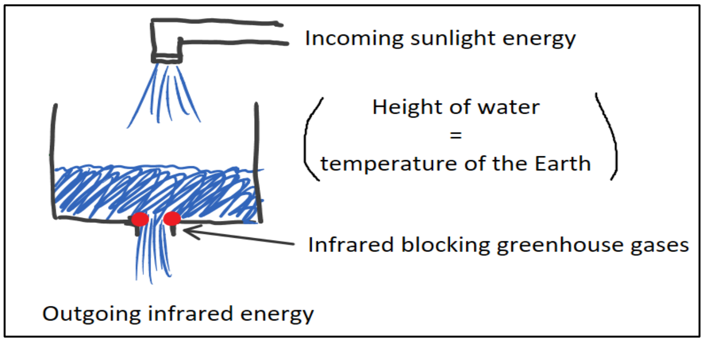
Model 3, is too simple of an energy-balance model to be used by climate scientists in their research, but it does indicate the effect that the Earth's naturally occurring heat trapping gases can have.

The Earth would be approximately 30 degrees colder, have an average temperature below zero, and probably be covered with ice if it were not for water vapour, CO_2, methane and the Earth's other naturally occurring greenhouse gases.

We owe the diversity of life on this planet, in part, to its natural greenhouse effect.

Extension Question:

Energy balance climate models can be simplified to a sink analogy.



Once the water level in the sink stabilizes, would the water flowing in equal the water flowing out?

Yes

If bits of spinach were blocking the sink's drain, would happen to the water level in the sink?

The water level would go up.

Is this analogous to what would happen to the Earth's temperature? Why do you think so?

Yes. By increasing greenhouse gases in the atmosphere, it is more difficult for heat to escape the Earth until the Earth's temperature goes up.

In the same way, the bits of spinach would make it more difficult for water to drain from the sink until the water pressure goes up by the height of the water increasing.