SPH4U: Measurement and Numbers

Recorder: __________________
Manager: __________________
Speaker: __________________
0 1 2 3 4 5

Measurements form the backbone of all science. Any theory, no matter how slick, is only as good as the measurements that support it. Without careful measurements, science becomes guess work, hunches and superstition.

A: The Meter Stick

Our most basic scientific tool is the meter stick. But, do you know how to use it? Really? For this investigation you will need one meter stick

1. **Observe.** Each member of your group will independently (and secretly!) measure the height of your table. Don't share the results until everyone has made the measurement. Record everyone's measured values here.

The number we read from a measurement device is the *indicated value*. The *readability* of a device is the smallest increment or change in a quantity that you can discern from the measuring device. The readability is sometimes called the reading error, but the term "error" is very problematic so we will avoid it. When we record a measurement, we should also record the readability with a statement like: "scale readable to ± 1.0 cm". The reported readability may vary from person to person. If you think you can estimate a reading between the lines of a scale, do so. Always record a measurement as carefully as you can.

2. **Reason.** What is the readability of your measuring device? Your estimation of the readability may be different from the others in your group, and that can be OK as long as you are using the device appropriately. Explain how you decided on your readability.

3. **Reason.** Now think about the height measurements your group has made. How do they compare with one another? Would you say, roughly speaking, that there is a lot of *uncertainty* or little uncertainty in your group's measurements? Explain.

The *true value* is the actual, ideal value for a quantity that we are trying to measure. The true value of a quantity is usually **never known:** in science this is simply not possible (welcome to science)! Through hard work and ingenuity, we try to get our measurements (our indicated values) closer to the true value, but there is always some *uncertainty* in this since we never get to know the true value!

4. **Reason.** Some groups find differences of about 0.5 cm amongst their height measurements. What are some suggestions for a future group to reduce the differences in their height measurements? (This is the ingenuity we mentioned.)

B: The Stopwatch

Now we will examine another common measuring device. You will need one stop watch

1. **Observe.** Measure the amount of time for the pencil to drop from a 1 m height. Write this reading as a number in decimal notation with units of seconds.

2. **Reason.** What is the readability of the stopwatch? Explain.

3. **Observe.** Perform the pencil drop seven times and record your data below.

4. **Reason.** Examine the individual measurements in your data above. You probably notice quite a bit of variation in them. What might be responsible for the spread in these values?

> Repeated measurements will usually produce a range or *distribution* of values. The size or width of this distribution is a measurement of the uncertainty in our result. One technique to find the width of the distribution is the *standard deviation,* but we will not use this in grade 12 physics. Instead, we will define the *uncertainty* (σ) by making a **very crude** estimation of the width (really the half-width) of the distribution:
>
> $$\sigma = (\text{high value} - \text{low value})/\text{number of measurements}$$
>
> Use the readability of your measuring device to limit the number of digits in your uncertainty result.

5. **Reason.** Based on your measurements, **w**hat is your best estimation of the true value for the time for the pencil drop? What is the uncertainty in this result? Show your work.

> When we present a calculated result based on our measurements we typically report two numbers, the best value (m) and the uncertainty (σ) written as: $m \pm \sigma$. This expression represents a *distribution of values* centred at the value m and with a width of σ on each side. We interpret this expression by saying, "m is our best estimation of the true value, but we wouldn't be surprised if the true value was as high as $m + \sigma$ or as low as $m - \sigma$."

6. **Reason.** When you computed your best value, the calculator likely displayed many digits. Use your uncertainty to help decide how to write the digits of your final result in the form $m \pm \sigma$. Write your result with uncertainty on a whiteboard.

> In the past you may have learned unhelpful rules about significant digits. There is just one proper rule: **only the uncertainty in a value determines which digits are significant (or meaningful).** Use the value of the uncertainty to limit the number of digits you write down.
> [e.g. your calculator reads 12.75834 and the uncertainty is ± 0.3, you write 12.8 m ± 0.3 m or 12.8(3) m]
>
> In other situations, like textbook problems or calculated results, we don't know the uncertainty and will use some simple rules for writing our numbers:
>
> - When recording results, just use three significant digits to avoid too much rounding error (3 digits determined by how it would be written in scientific notation. e.g. your calculator reads: 1 056 428, you write: 1 060 000 or 1.06×10^6).
> - For middle steps in calculations, keep a fourth or fifth *guard digit* to help reduce the amount of rounding error.
> - Out of convenience, we will write 5 instead of 5.00, with the understanding that it has three significant digits

7. **Calculate.** Make a calculation to predict the time for the pencil to drop (use the equation $\Delta y = v_1\Delta t + \frac{1}{2}a\Delta t^2$ and $a = 9.8$ m/s^2).

> A fundamental process in science is to decide whether two results "agree" with one another. We will adopt a simplified decision rule for this. Two results *agree* with one another if one lies within the uncertainty distribution of the other, or if their distributions overlap. In more advanced studies, you will refine and greatly strengthen this crude rule.

8. **Evaluate.** Does this calculated value agree with your measured result? Explain.

SPH4U Homework: Measurement and Numbers Name:

A: The Pebble Drop

It is sunset. You and a friend walk on to a bridge that passes over a river. After gazing off into the distance and into each other's eyes you both arrive at the same question: How high are we above the water? Luckily you have your smartphone with a timer app. Your friend finds a few rocks which he releases ($v_1 = 0$). You time the fall until you see them splash in the water below. Your data is shown below.

1.73 s	1.79 s	1.82 s	1.69 s	1.81 s	1.77 s	1.74 s

1. **Calculate**. Based on your measurements, find the best value for the time for the rock to fall. Express your result in the form $m \pm \sigma$ with an appropriate number of digits. Show your work.

2. **Calculate.** Use your time result to calculate the distance the rocks fell (use $\Delta y = v_1\Delta t + \frac{1}{2}a\Delta t^2$). Your result from the calculation should use one or two guard digits. When write your final statement, use three digits.

D: Mathematical Representation

Describe steps, complete equations, algebraically isolate, substitutions with units, final statement

We don't know the uncertainty in your final calculated result. Since the time value you used in the calculation has an uncertainty, we expect the distance result to also have an uncertainty. There are sophisticated techniques to find this uncertainty, but we will not use these in grade 12 physics. Instead, make a simple, somewhat educated, estimation (guess!) for the uncertainty in the final quantity. For example, do you think your result is reliable to 1%, 5%, or 10% of its value? Decide and use that to estimate an uncertainty. This is an extremely crude technique whose purpose is to make you aware that there **are** uncertainties in scientific calculations and that these uncertainties guide our scientific decision making.

3. **Evaluate.** Do you think your calculated result is reliable to within 1% of its value? Or maybe 10%? Or maybe in between? State your estimated uncertainty and use it to write your final result with an uncertainty. (For example: estimated uncertainty is 10%, Δy = 27.5 m ± 2.8 m)

4. **Evaluate**. You use your smart phone to look-up the height of the bridge and find a result of 15.1 m. Does your calculated result agree with this value? Explain.