# NEWSLETTER

**ONTARIO ASSOCIATION OF PHYSICS TEACHERS**
(An Affiliate of the American Association of Physics Teachers)
February, 2004

## Why Does Gravity Slow Time?

**By Richard Epp**

Imagine a pair of twins, Alice and Bob, who will live to exactly the same age. Rather than giving this age in years, which might be confusing in what follows, let's say each will live for one billion heart beats, and their hearts beat at 60 beats per minute. Alice, a hurricane hunter by trade, has become bored with Earth's puny storms and has moved to Jupiter to chase its Great Red Spot, a centuries old cyclone of truly mammoth proportions.

Now gravity is stronger on Jupiter than on the Earth, one consequence being that Alice weighs more. But more interestingly, Albert Einstein's theory of general relativity—his theory of space, time and gravity—says that, due to the lower gravitational potential on Jupiter than on Earth, time as experienced by Alice is moving more slowly *relative* to time experienced by Bob back on the Earth. What does this mean? First, the word 'relative' is crucial here: it means that as far as Alice is concerned, nothing in her own experience indicates to her that time is moving more slowly. The point is, more slowly *relative to what*? Alice herself feels nothing out of the ordinary, for instance her heart still beats at 60 beats per minute according to *her* wristwatch. It is only when Alice and Bob *compare* their experiences of the passage of time that they notice something very strange.

For example, when they speak with each other over the satellite link, Bob notices that Alice's voice is a bit deeper and she is speaking more slowly—exactly like a tape recording replayed at a slightly slower speed. But Alice does not feel that she is speaking slowly, or thinking slowly, or anything else for her is happening more slowly. And from Alice's point of view, she notices that Bob's voice is *higher* pitched than she remembers, and he is talking (and thinking, and doing everything else) a bit *faster*—exactly like a tape recording played back at a faster speed. More to the point, when Bob puts the microphone up to his heart, Alice hears it beating at *faster* than 60 beats per minute according to *her* wristwatch (and *her* heart); conversely, Bob hears Alice's heart beating more slowly than his. Both agree that Alice could return to Earth before her billion heart beats run out and attend Bob's funeral.

How is this possible? Why does gravity affect the rate at which time moves? One of the simplest ways to understand this begins with Einstein's equation $E = mc^2$, which says that mass is a form of energy. As a consequence of this, Einstein reasoned that it would be possible to build a perpetual motion machine (and thus get energy from nothing) *unless gravity slows time*. This machine consists of a vertical conveyor belt stretched between two pulleys which, to keep things simple, we imagine to be frictionless. Attached to the belt are a number of identical, equally spaced buckets, each of which contains a single atom of mass $m$. Now suppose each of the atoms on the left side of the belt have absorbed a photon of energy $\Delta E$. These excited atoms have more energy, *and hence more mass*. The increase in mass is $\Delta m = \Delta E/c^2$. With the whole apparatus in the gravitational field of the Earth, the force of gravity will be greater on the excited atoms (each with mass $m+\Delta m$) than the ground state atoms (each with mass $m$), resulting in a net force causing the conveyor belt to begin to rotate counter clockwise.

To keep the conveyor belt moving we arrange for the excited atoms to emit a photon of energy $\Delta E$ as they reach the bottom, leaving them in their ground state (with mass $m$). We then use a mirror to direct the emitted photons up to the top, and use these photons to re-excite the ground state atoms as they arrive at the top of the conveyor belt. *Assuming* the energy of the photons received at the top is $\Delta E$ (the same energy they had at the bottom), clearly this is a perpetual motion machine: the heavier, excited atoms will always be on the left, the lighter ground state atoms on the right, resulting in a constant, net tendency for the belt to rotate counter clockwise. We could use this tendency to make our machine do useful work. Energy for nothing! All the world's energy problems solved!

Of course, this must be impossible. But where is the mistake in our reasoning? A bit of thought reveals that the mistake is in assuming that the energy of the photons received at the top is $\Delta E$. In fact it must be *less* than $\Delta E$; call it $\Delta E'$. How much less? In being lowered from the top to the bottom (through a height $h$, say), each excited atom yields up its gravitational potential energy of $(m+\Delta m)gh$, where $g$ = 9.8 m/s$^2$. However, for each excited atom thus lowered, there is a ground state atom raised, which uses up an amount of energy $mgh$. The net gain in energy is the difference: $\Delta mgh$. To avoid this net gain (energy for nothing), it must be true that $\Delta mgh$ is precisely equal to the energy lost by the photon as it travels from the bottom to the top, i.e. $\Delta E - \Delta E' = \Delta mgh$. One may imagine the photon losing energy as it climbs against the Earth's gravitational field much like a rock thrown upward loses kinetic energy as it slows down, the main difference being that the photon does not slow down; it always moves at the speed of light. Using $\Delta m = \Delta E/c^2$, the previous equation reads: $\Delta E - \Delta E' = \Delta Egh/c^2$. Dividing both sides by $\Delta E$ and rearranging terms we get: $\Delta E' / \Delta E = 1 - gh/c^2$. Finally, we also know that the energy of a photon is proportional to its frequency (the proportionality constant being Planck's constant). So we can rewrite our equation as: $f' / f = 1 - gh/c^2$, where $f$ (resp. $f'$) is the frequency of the photon at the bottom (resp. top). Since $1 - gh/c^2$ is less than one it says that the frequency of the photon received at the top (as measured by a clock at the top) is *less* than the frequency of the photon that left the bottom (as measured by a clock at the bottom).

This is a very strange result. To appreciate why, it will help to switch from the particle picture of light (photons) to the wave picture, and recall that electromagnetic waves can be produced by an oscillating electric charge. Our result says that if Bob is standing on the surface of the Earth holding an electric charge in his hand, and waving his hand back and forth once per second ($f$ = 1), then Alice, standing at the top of a tower of height $h$, will receive electromagnetic waves of frequency $f' < 1$. Said more directly, wave crests of light leaving Bob's hand once every second (one second according to *his* wristwatch), will arrive at Alice's position at a rate of *less than* once every second (one second according to *her* wristwatch).

Wouldn't this require the number of wave crests *between* Bob and Alice to be continually increasing with time? Can't be! With a fixed distance $h$ between the two, such a continual bunching up of wave crests would require the wavelength of the light (distance between crests) to be getting smaller and smaller as time goes on. Since the speed of light is always the same, smaller wavelength means higher frequency, which means Alice would be seeing light of higher and higher frequency as time goes on. This is definitely not happening; Alice sees light of *constant* frequency, $f'$. (Also notice that $f' < f$ means the wavelength is longer for Alice than it is for Bob; the wave train 'stretches out' as it moves upwards) But if there is no bunching up of wave crests, how do we explain that Alice sees fewer than one wave crest per second according to her wristwatch (equivalently, more than one second of Alice-time elapses between crests), *unless time itself is moving more quickly at Alice's location than at Bob's*? This is the effect of gravity on time!

Here is an alternative way to see this. As Bob is waving his hand back and forth, a wave crest leaves the electric charge in his hand each time it reaches the right end of its back and forth arc. Any other light illuminating Bob's hand (that allows Alice to see Bob) will travel to Alice at the same speed as the wave crests of light leaving the electric charge in his hand. This means that when Alice looks down at Bob she must see his hand waving back and forth at the *same* frequency that the crests of light from the electric charge are reaching her, i.e. at the frequency $f' < 1$. Thus, Alice sees Bob waving his hand at a rate of *less* than once per second according to *her* wristwatch. Bob appears to be waving his hand in slow motion! Indeed, everything about Bob and his surroundings on the ground will appear to Alice to be happening in slow motion. Conversely, Bob will see Alice in fast motion, like a video on fast forward.

Notice, also, the longer Alice waits at the top of the tower, the greater the difference that will accumulate between the number of seconds (and heart beats) she experiences and the number Bob experiences. Eventually climbing back down from the tower and comparing notes, both will agree that more seconds (and heart beats) have elapsed for Alice than for Bob. Alice will be older than Bob because she has spent time in a place (height $h$ above the Earth) where time itself moves more quickly *relative* to time on the Earth's surface! In the case of Alice going to Jupiter, the gravitational potential will at first increase as she leaves the Earth's surface (her time will move more quickly relative to Bob's time, as just discussed), and will continue to increase as she travels away from the Sun towards Jupiter. But eventually the gravitational potential will decrease to a value *lower* than that on the Earth's surface as she descends into Jupiter's very strong gravitational field. On Jupiter her time will be moving more *slowly* relative to Bob's time, as described at the beginning of this essay.

So how big is the effect of gravity on time? Very tiny, at least as far as we are able to experience here on the Earth. For example, taking $h$ = 100 metres gives $f' / f = 1$

$- gh/c^2$ 0.99999999999999. This means that Bob's clock is running 99.999999999999% as fast as Alice's, meaning we would have to wait a very long time for a noticeable difference in age to accumulate. Nevertheless, the effect has been measured using very accurate atomic clocks and the results are in excellent agreement with our formula. Indeed, the Global Positioning System (GPS), which relies on very accurate atomic clocks both on the Earth and carried by satellites high above the Earth, must account for this effect in order to work with the accuracy it does. It should be emphasized, however, that the effect is quite noticeable in more interesting parts of the universe than our solar system. A black hole is an extreme example, where gravity is so strong at the event horizon that time is slowed to a stop relative to anyone outside the horizon! But this is another story...

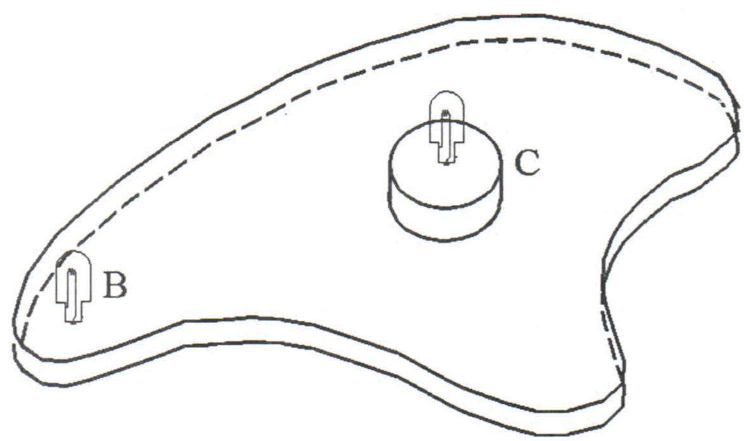
*Richard is the outreach Program Coordinator at **Perimeter Institute for Theoretical Physics**. His PhD degree is in theoretical physics, specializing in general relativity and quantum gravity. For information on research and outreach at Perimeter Institute please see www.perimeterinstitute.ca*

## The Demonstration Corner

### Motion of the Centre of Mass

by Patrick Whippey, Department of Physics & Astronomy,
The University of Western Ontario.

Do we really believe Newton's Laws? This demonstration was born many years ago when a perceptive student challenged the assertion that a body free to move always rotates about its centre of mass. This demonstration requires an air table.




The figure shows a piece of plastic about 0.5 cm thick. It is about 20 cm long and 15 cm wide, and has an irregular shape. Mounted on the plastic are a battery and two bulbs. We use a pair of flat batteries of the kind used in small electronic devices together with some light-emitting diodes (LEDs), but a pair of AA batteries plus a pair of flashlight bulbs will serve just as well. Bulb C is located at the centre of mass, while bulb B is way off to one side.

We first cover bulb B. We give the "amoeba" a push and watch it travel over the air table. Bulb C travels in straight lines, changing direction only when the "amoeba" bounces off the edges of the air-table.

Then we cover bulb C. Bulb B travels in complex curved paths, making striking patterns of light. Finally, we expose both light bulbs. We can then see that bulb C travels in straight lines, while bulb B makes circles around it. This is a striking demonstration that the centre of mass travels in a straight line while all other points rotate around the centre of mass. This demonstration is best seen in the dark.

If you use LEDs, they may require a current-limiting resistor of about 150 ohms in series.

*Column Editor: Ernie McFarland, Physics Department, University of Guelph, Guelph, Ontario, N1G 2W1*
*Email: elm@physics.uoguelph.ca*

*Submissions describing demonstrations will be gladly received by the column editor.*

## Conference Reminder!

This year's conference is hosted by the University of Ontario Institute of Technology (UOIT) in Oshawa, and will be held May 27 – 29, 2004. Presenters are still welcome, and invited to forward information (see Dec. letter for details) to Elzbieta Muir (emuir@sympatico.ca) by April 30.

Participants may find additional information regarding the conference and registration at the following web site: www.uoit.ca/schoolofscience/OAPT2004.

## Call for Conference Resources

This year's conference will run the "Share & Exchange Resources" Session on Saturday, May 28 to encourage collegiality and to facilitate the exchange of high school physics resources. Please participate actively in the conference by contributing your favourite worksheet, test, or exam. Your resource should be classroom proven, and come complete with an answer key. It should be neat and have a pleasant layout. Please bring photocopies (30 or more) to the Session for distribution. If photocopying is not possible, then please bring one copy of your resource to the Session; you will be asked to forward it electronically to interested teachers after the conference. To assure that the Session runs smoothly, you are asked to forward, *at your earliest convenience*, to Elzbieta Muir (email: emuir@sympatico.ca) the following:

a) your name, school, and city;
b) the title of your worksheet/test/exam; and
c) the course and textbook title in which your resource is used.

The first 10 teachers will be recognized and listed in the conference program. Deadline: **April 30, 2004.**

## Annual Election, 2004

The OAPT membership at large is asked to nominate members for the following three positions of the OAPT Executive Committee for 2004-2005:

- **Vice-President** (chairman of the Section, succeeds the President the following year),
- **Secretary-Treasurer** , and
- **member-At-Large**.

Please submit your nominations to Elzbieta Muir (emuir@sympatico.ca) by e-mail by April 30, 2004.

## Membership Matters!

Join the Ontario Association of Physics Teachers! Members receive a Newsletter and reduced registration rates at the annual conference.

As well, from time to time, the Association makes available special resources. Examples have included reprints of **"Demonstration Corner"** articles from the **Newsletter**, and the videotape, **"The Physics of Dance,"** from a presentation at one of the annual conferences.

To become a member of the OAPT, send a cheque for $8 (or a multiple thereof) payable to OAPT to:

**Ernie McFarland**
**OAPT Membership Coordinator**
**Department of Physics**
**University of Guelph**
**Guelph, Ontario**
**N1G 2W1**